Power Dependence of HTS Disk-Resonator Quality Factor

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***Abstract*—The simple closed-form model of nonlinear response of a high-temperature superconducting disk resonator on microwave power is proposed. The model is based on superconducting film nonlinearity and describes the dependence of unloaded quality factor on the incident power. The specified normalizing power is used as the only fitting parameter. Good quantitative agreement of modeled and measured data has been obtained. The results observed exhibit the kinetic nature of the nonlinearity of the disk resonator on an $LaAlO_3$ substrate at $T = 60$ K and more complicated, presumably thermal, heating nature of the nonlinearity at lower temperature.**

***Index Terms*— High-temperature superconductors, modeling, nonlinearities, superconducting resonator.**

I. Introduction

The rotational symmetric mode (TM_{010}) of a superconductor disk resonator is used to perform a high-quality resonator [1], [2], particularly for increasing power-handling capability of planar high-temperature superconductor (HTS) filters [3]–[5]. The simple model of a nonlinear response of an HTS transmission line and a microstrip resonator was recently suggested and verified [6]. The goal of this paper is to apply the model [6] to the HTS disk resonator in order to obtain, in closed form, the dependence of the quality factor of the disk resonator with the mode TM_{010} on the incident power. The phenomenological parameter used in [6] for modeling nonlinear properties of the superconducting film can be extracted from the experimental data of [3] and [4], and some useful comparison can be done between the microwave nonlinear characteristics and the critical current of the film measured at direct current.

II. Field Distribution and Quality Factor of the Resonator in a Linear Approach

Fig. 1 shows the field distribution in the resonator for the mode TM_{010}.[1] The field distribution is given by the following equations:

$$\vec{E}(r) = \vec{e}_z E_m \cdot J_0(k_r r)$$
$$\vec{H}(r) = \vec{e}_\varphi H_m \cdot J_1(k_r r) \qquad (1)$$

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[1] The mode TM_{010} was named in [2] as the mode TM_{020} by mistake.

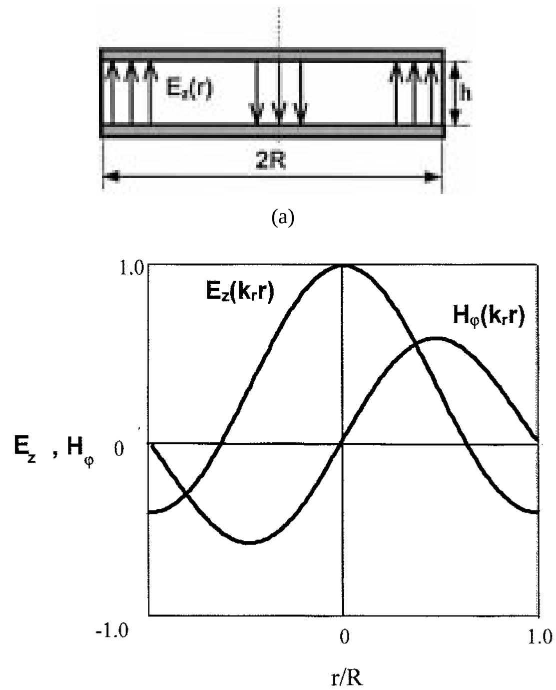

Fig. 1. Electrical- and magnetic-field distribution in the circular disk resonator for TM_{010} mode.

where

$$k_r = \frac{3.83}{R} \qquad (2)$$

$$H_m = E_m \cdot \left(-\frac{k_r}{i\omega\mu_0}\right). \qquad (3)$$

$J_0(k_r r)$ and $J_1(k_r r)$ are Bessel functions, and R is the radius of the resonator.

The current density is determined in accordance with (1) as

$$\vec{j}(r) = \vec{e}_r H_m \cdot J_1(k_r r). \qquad (4)$$

The quality factor of the resonator in a linear approach is determined as follows:

$$Q = \frac{120\pi}{R_{\mathrm{sur}}} \cdot g \qquad (5)$$

where g is the geometric factor which can be found by averaging oscillating and dissipated power in the resonator [5]

$$g = \frac{2\pi \int_v |\vec{H}(r)|^2\, dv}{\lambda_0 \int_s |\vec{H}(r)|^2\, ds} \qquad (6)$$

$R_{\rm sur}$ is the surface resistance of the superconducting film, and λ_0 is the wavelength in the free space. Integration in (6) should be done over the volume of the resonator and over the surface of the superconducting plates. The dielectric loss is not taken into account. In general, the surface resistance can depend on coordinates, which can be taking into account by the integration of function $f(r, \varphi) = R_{\rm sur}(r, \varphi)\cdot|\vec{H}(r)|^2$ over the surface. We assume that the surface resistance is constant within the boundaries of the superconducting plates.

Calculation of the integrals gives

$$g = \frac{\pi h}{\lambda_0}. \tag{7}$$

Next, we consider the quality factor determined by (5) as the unloaded quality factor Q_U. It is useful to remind that Q_U can be expressed by the ratio of oscillating power to dissipated power in the resonator

$$Q_U = \frac{P_{\rm oscil}}{P_{\rm diss}}. \tag{8}$$

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III. Oscillating Power in the Resonator

Let us consider the resonator as a symmetrical two-port. In this case, the reflection and transmission coefficients at resonant frequency can be found as

$$\begin{aligned} |S_{11}|^2 &= \frac{(Q_E/Q_U)^2}{(1+Q_E/Q_U)^2} \\ |S_{12}|^2 &= \frac{1}{(1+Q_E/Q_U)^2} \end{aligned} \tag{9}$$

where Q_U is the unloaded quality factor of the resonator determined by the geometry of the resonator and by properties of the superconducting film, and Q_E is the external quality factor of the resonator defined by coupling the resonator with the external transmission lines.

The power dissipated in the resonator can be expressed through the incident power[2]

$$\frac{P_{\rm diss}}{P_{\rm incid}} = 1 - |S_{11}|^2 - |S_{12}|^2 = \frac{2(Q_E/Q_U)}{(1+Q_E/Q_U)^2}. \tag{10}$$

Then, using (8) and (10), we obtain a formula giving relation between the oscillating power in the resonator and the incident power in the linear approach

$$\frac{P_{\rm diss}}{P_{\rm incid}} = \frac{2\cdot Q_E}{(1+Q_E/Q_U)^2}. \tag{11}$$

IV. Nonlinear Dissipation in the Resonator

Using the main idea of the model [6], one may describe the surface resistance of the film as a function of the alternating field amplitude

$$R_{\rm sur}(r) = R_{\rm sur,\,0}\cdot\left[1 + \frac{H_m^2\,\cos^2\,\omega t\cdot J_1^2(k_r r)}{H_0^2}\right] \tag{12}$$

[2]The incident power $P_{\rm incid}$ is determined as the available input power in a 50-Ω transmission line terminated in a matched load.

where H_0 is a phenomenological characteristic of the nonlinearity of the superconducting film. It should be stressed that the phenomenological parameter H_0 is not directly connected with the critical magnetic field H_{c1}. We do not discuss here the nature of the nonlinearity. The nonlinear behavior of the HTS films can arise from a kinetic process, the microwave heating can be considered as a competitive mechanism, and the local defects of the film can also influence the nonlinear characteristic of the resonator under microwave power [5]. The parameter H_0 comprises all possible mechanisms and is assumed to be extracted from experimental data. Now, we can calculate the power dissipation in the resonator, taking into account the nonlinear increase in the dissipation. The dissipated power can be calculated upon integrating over the surface of the superconducting plates, as in (6), and with respect to time over the period of microwave oscillations [6]. As a result, one obtains

$$P_{\rm diss,\,eff} = P_{\rm diss}\cdot\left[1 + \frac{H_m^2}{H_0^2}\cdot\kappa\right] \tag{13}$$

where

$$\kappa = \frac{3}{4}\cdot\frac{\displaystyle\int_0^R J_1^4(k_r r)r\,dr}{\displaystyle\int_0^R J_1^2(k_r r)r\,dr}. \tag{14}$$

Calculation for $k_r = 3.83$ yields

$$\kappa = 0.189. \tag{15}$$

Now, taking into account equation (8), we can introduce the effective unloaded quality factor of the resonator substituting $P_{\rm diss}$ by $P_{\rm diss,\,eff}$ from (13)

$$Q_{U,\,\rm eff} = \frac{Q_U}{1 + \dfrac{H_m^2}{H_0^2}\cdot\kappa}. \tag{16}$$

That can be rewritten as follows:

$$\frac{Q_U}{Q_{U,\,\rm eff}} = 1 + \kappa\cdot\frac{P_{\rm oscil}}{P_o} \tag{17}$$

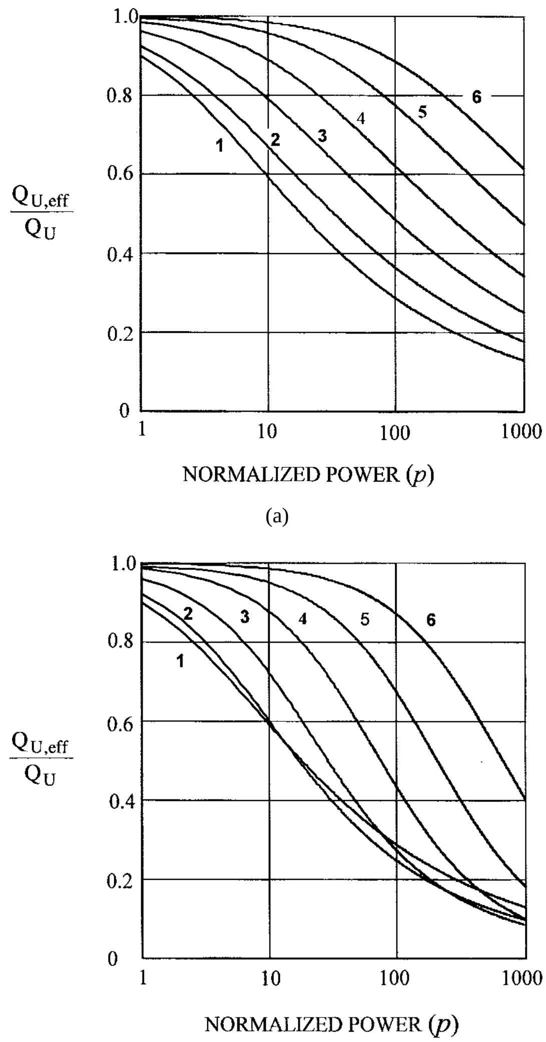
where $P_{\rm oscil}$ is the oscillating power in the resonator with the amplitude of the microwave magnetic field H_m, and P_0 is the effective oscillating power, which would be in the resonator if the amplitude of the magnetic field is H_0.

V. Effective Unloaded Quality Factor as a Function of the Incident Power

Returning to the equation for the oscillating power (11), we should replace Q_U in (11) by $Q_{U,\,\rm eff}$. Then, combining (11) and (17), one obtains

$$\frac{Q_U}{Q_{U,\,\rm eff}} = 1 + 2\kappa\cdot\frac{Q_E}{\left(1+\dfrac{Q_E}{Q_{U,\,\rm eff}}\right)^2}\frac{P_{\rm incid}}{P_o}. \tag{18}$$

From (18), the effective unloaded quality factor $Q_{U,\,\rm eff}$ can be found as a function of $P_{\rm incid}$.



(a)

(b)

Fig. 2. Dependence of effective unloaded quality factor $z = Q_{U,\mathrm{eff}}/Q_U$ on the normalized power p for different coupling. (a) $A = 1$ (1), 3 (2), 10 (3), 30 (4), 100 (5), 300 (6): weak coupling. (b) $A = 1$ (1), 0.3 (2), 0.1 (3), 0.03 (4), 0.01 (5), 0.003 (6): strong coupling.

It is convenient to introduce the following notations:

$$z = \frac{Q_{U,\mathrm{eff}}}{Q_U} \quad A = \frac{Q_E}{Q_U} \quad p = \frac{P_{\mathrm{incid}}}{P_N} \tag{19}$$

where

$$P_N = \frac{P_0}{Q_U}. \tag{20}$$

The introduced notations have the following sense:

- z characterizes decreasing the $Q_{U,\mathrm{eff}}$ under the influence of the incident power;
- A measure of coupling the resonator with the external circuits;
- p normalized incident power.

The normalizing power P_N will be discussed later. Using the notations, (18) can be rewritten as follows:

$$\frac{1}{z} = 1 + 2\kappa \cdot \frac{A}{\left(1 + \dfrac{A}{z}\right)^2} \cdot p. \tag{21}$$

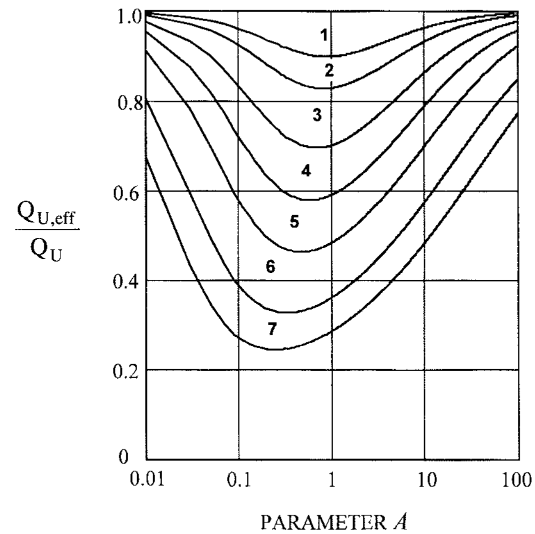


Fig. 3. Dependence of effective unloaded quality factor $z = Q_{U,\mathrm{eff}}/Q_U$ on the coupling parameter $A = Q_E/Q_U$ for different values of normalized incident power: $p = 1$ (1), 2 (2), 5 (3), 10 (4), 20 (5), 50 (6), 100 (7).

Fig. 2 shows the result of solution to (21), where $z = Q_{U,\mathrm{eff}}/Q_U$ is presented as a function of p for the cases of: 1) weak coupling of the resonator with the external circuits ($A > 1$) and 2) strong coupling ($A < 1$). Fig. 3 shows the dependence of $z = Q_{U,\mathrm{eff}}/Q_U$ on the coupling parameter A for different values of the normalized incident power p. One may see from the figures that decreasing the effective unloaded quality factor of the resonator under the influence of the incident power sufficiently depends on the coupling of the resonator with the external circuits.

VI. The Normalizing Power

The normalizing power P_N is determined by (20). As was defined, the phenomenological parameter P_0 can be presented as

$$P_0 = \tfrac{1}{2}\,\omega\mu_0 H_0^2 \cdot h \cdot 2\pi \int_0^R J_1^2(k_r r) r\, dr \tag{22}$$

which can be rewritten as

$$P_0 = 120\pi H_0^2 \cdot \frac{2\pi}{\lambda_0} \cdot h \cdot 2\pi R^2 \cdot 0.595. \tag{23}$$

At the same time, one has

$$Q_U = \frac{120\pi}{R_{\mathrm{sur}}} \cdot \frac{\pi h}{\lambda_0}. \tag{24}$$

Combining (23) and (24), one obtains

$$P_{N,k} = 7.5 R_{\mathrm{sur}} (H_0 R)^2. \tag{25}$$

For a rough estimation of the normalizing power, we may suppose that

$$H_0 = j_{c,\mathrm{vol}} \cdot \lambda_L \tag{26}$$

where $j_{c,\mathrm{vol}}$ is the volume critical-current density of the superconducting film measured at dc, and λ_L is the London penetration depth of the film. Equation (26) cannot be considered as well grounded from the physical point of view. It

TABLE I
PARAMETERS OF THE RESONATORS [3], [4]

	Resonator No. 1 [3]	Resonator No. 2 [4]	Resonator No. 3 [4]
Radius R, mm	10.5	18.75	18.75
Dielectric thickness h, mm	1.0	0.5	0.5
Resonant frequency f, GHz	3.5	2	2
T, K	60	60	30
Q_U (experimental)	60,000	89,500	880,000
L_d (experimental)	3	4	1
Coupling parameter A	0.07	0.047	0.263
Nomalizing power P_N as a fitting parameter	3.5	3.5	0.005

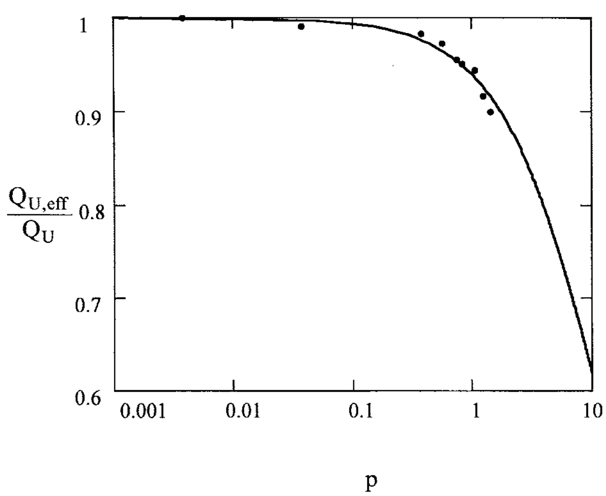


Fig. 4. Measured (points) and simulated (solid line) data for resonator 1 [3].

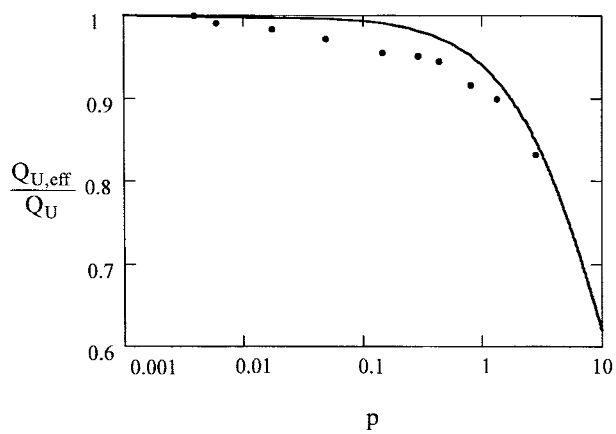


Fig. 5. Measured (points) and simulated (solid line) data for resonator 2 [4].

reflects only the kinetic nature of the nonlinearity of the film. The dielectric loss was not taken into consideration thus far. In the case when the dielectric loss is comparable with the loss in HTS conductors of the disk resonator, (21) may be used in the following form:

$$\frac{1}{z} = 1 + 2k \cdot \frac{A \cdot L_d}{\left(1 + \dfrac{A}{z}\right)^2} \cdot p \tag{27}$$

where

$$L_d = 1 + \frac{Q_S}{Q_D} \tag{28}$$

Q_S is the quality factor determined by HTS film parameters and calculated by (24), and Q_D is the quality factor arisen from the dielectric loss in the resonator [5].

VII. Some Numerical Estimations

Let us use the results of measurements of characteristics of the disk resonators on an $LaAlO_3$ substrate [3], [4]. The parameters of the resonators are presented in Table I. The modeling characteristics (see Figs. 4–6) simulated by (27) differs by coupling parameter A and dielectric dissipation factor L_d (see Table I). The experimental results are presented in Figs. 4–6 in dimensionless form, using suitable fitting parameters P_N (see Table I). The results of modeling are very close to the experimental results.

VIII. Discussion

The results of calculation of the nonlinear performance of the resonators in a comparison with experimental data presented in Figs. 4–6 support the model proposed. The main point of the model is the nonlinear description of the surface resistance (12) and the use of the normalizing power P_N

TABLE II
PERFORMANCE OF THE HTS FILMS IN THE RESONATORS [3], [4]

	Resonator No. 1 [3]	Resonator No. 2 [4]	Resonator No. 3 [4]
T, K	60	60	30
Surface resistance of the YBCO film R_{sur}, Ohm	$1.3\cdot10^{-4}$	$4.4\cdot10^{-5}$	$7.85\cdot10^{-6}$
London penetration depth λ_L, μm	0.23	0.23	0.17
Critical current j_c, A/cm^2	$1.5\cdot10^{6}$	$2.5\cdot10^{6}$	$5\cdot10^{6}$
Power $P_{N,k}$ calculated by formula (25), W	4.13	3.845	0.015
$P_N/P_{N,k}$	0.847	0.91	0.33

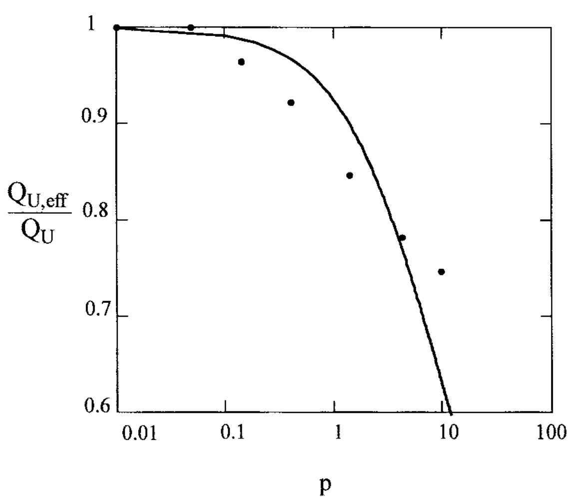


Fig. 6. Measured (points) and simulated (solid line) data for resonator 3 [4].

in (27) as a fitting parameter. The parameters P_N for two different resonators measured at $T = 60$ K are roughly the same. For the resonator measured at $T = 30$ K, the fitting parameter P_N is much smaller.

Let us evaluate the normalizing power $P_{N,k}$ for three experimental resonators using (25). The film parameters λ_L and j_c are chosen as typical ones [3]–[5], [7], [8] ($l_L(0) = 0.15$ mm), the surface resistance for each film was extracted from the experimental value of the unloaded quality factor using (24). The results are presented in Table II. The last row of Table II shows the ratio $\eta = P_N/P_{N,k}$.

Evidently, the ratio η for resonators 1 and 2 measured at $T = 60$ K is close to the unity.

Good qualitative agreement between modeled and measured data for two disk resonators on $LaAlO_3$ substrates at $T = 60$ K supports the idea that the source of the nonlinearity of the HTS resonator under the high-level incident power is the nonlinear surface resistance in form (12). The normalizing power $P_N \approx P_{N,k}$ used in the nonlinear equation (27) can be estimated by the phenomenological magnetic field H_0, which seems to be determined by dc parameters of the HTS films [9]. That means that the nonlinear performance of the resonator arises from the kinetic effects in the HTS films.

For resonator 3 measured at $T = 30$ K, the model developed does not fit the experimental results with high accuracy. The ratio η is sufficiently less than 1. Using (25) as the definition of the normalizing power P_N in the physical sense, one can propose that the phenomenological parameter H_0 and, as a consequence, the critical current density j_c are sufficiently decreased at low temperature. It is difficult to find a reason for this phenomenon in a framework of kinetic processes in the film. A realistic explanation is the suggestion of a complicated nature of this effect at low temperature. It is well known that the thermal conductivity of dielectric substrates used for HTS film growing is strongly temperature dependent [10]. The thermal boundary resistance at the film/substrate interface is also temperature dependent [11] and increases with decreasing temperature. As a consequence, the dissipated power in the resonator is significantly determined by thermal-flow propagation in the HTS film–substrate system. A deterioration of the heat removing at low temperature leads to the HTS film heating and changing film parameters. Thus, the higher sensitivity to the incident microwave power at low temperature may be accounted for by taking into consideration the thermal process in the HTS film–substrate system.

The results observed exhibit the kinetic nature of the nonlinearity of the resonator on an $LaAlO_3$ substrate at $T = 60$ K and more complicated, presumably thermal, heating nature of the nonlinearity at lower temperature.

IX. Conclusion

The simple closed-form expressions allow one to evaluate decreasing the unloaded quality factor of the high-quality HTS planar disk resonator against the incident power. Only one fitting parameter P_N is used. This parameter includes all mechanisms of the nonlinearity of the HTS film and can be extracted from the experimental data. In some cases, the parameter P_N can be estimated by the HTS film parameters including critical current density measured at direct current.

The special investigations of a dependence of the effective unloaded quality factor $Q_{U,\mathrm{eff}}$ on the incident power in combination with characteristics of the HTS film and the substrate can give information about the actual value of P_N and about the nature of the nonlinearity of the superconducting films at microwaves.

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